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The influence of the logarithmic spiral on the multivariate analysis of coiled shells

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Abstract

The logarthmic spiral structure of the conch of coiled ammonites introduces a complication with respect to the multivariate analysis of the variability of certain shell- dimensions by the technique known as "principal components". This is the outcome of the fact that the lateral form of the shell is geometrically determined by the properties imposed by the logarthmic spiral. The problem is quite general and is of consequence for all organisms that grow according to the geometry of the logarithmic spiral.

Introduction and statement of the problem

Ammonite shells are coiled in close approximation to the geometry of the logarithmic spiral. This imposes a constraint on the results yielded for the shape analysis of the conch based on measurements made on the shell in lateral aspect if the set of variables are part of the spiral growth pattern (Klein, 1926, pp. 171-173). One of the inescapable outcomes of the extraction of latent roots and vectors of the matrix of covariances of such data is that the square symmetric matrix is virtually of unit rank. Hence, all possible selections of variables, such as the width of the umbilicus, the maximum diameter of the conch, breadths of whorls are rigidly constrained; clearly, this renders a standard principal component analysis of such data redundant.

In order to illustrate how this constraint influences a result, two examples are briefly reviewed in the present note.

Discoscaphites, a heteromorphic ammonite genus

As an example we consider a, a simple analysis of four lateral measurements of the coiled section of shells of Maastrichtian macroconchs of *Discoscaphites conradi (*Morton*)*, to wit, maximum diameter, two whorl dimensions and umbilical width. This manoeuvre yielded the latent roots (Landman and Waage, 1993, p. 248).

Latent roots	(1) 1.0586	(2) 0.0125	(3) 0.0005	(4) 0.0002
Percentages	98.76	1.17	0.05	0.02

There are three very small latent roots of which two are almost zero, whereas the largest latent root accounts for almost all of the variability. In approximate terms, the rank of the covariance matrix from observations in the plane of coiling is almost of unit rank. What does this signify? It is expressing the fact that the logarithmic growth spiral locks the variability in a firm grip and the analysis is not providing any information of taxonomic or growth-variability significance.

Schloenbachia, a morphologically complicated genus of ammonites

The Lower Cretaceous ammonite genus *Schloenbachia* is remarkable for great variability in apertural shape characteristics. This in turn means that a standard taxonomical characterization of forms into "palaeo-species" is often not a logically unchallengeable exercise.. This is a somewhat more complicated problem than that posed by *Discoscaphites*.

A sample of 17 well preserved shells from the Lower Cenomanian of Bed 30 of the Bezakty section of Mangyschlak (Kazachstan) were subjected to a principal component extraction of the covariance matrix based on four variables observed on the apertural surface, to wit:

1 - breadth of the conch, an apertural variable
2.- height of the last whorl
3.- diameter of the umbilicus
4- maximum diameter of the shell

The variables 2-4 are lateral variables and hence can be expected to be logarthmically constrained in some manner. Figure 1 demonstrates the locations of the distance-measures on a mature specimen of *Schloenbachia varians* (Sowerby, 1817) from the Lower Cenomanian deposits at Wiltshire. U. K.

The latent roots for covariances are

(1) 2.30747 (2) 1.14871 (3) 0.32269 (4) 0.22113

Percentages of the total variance

(1) 57.6868 (2) 28.7176 (3) 8.0673 (4) 0.5282

Latent vectors by columns

	1	2	3	4
1.	0.0304	0.9114	0.2532	-0.3230
2.	0.5832	-0.2724	-0.0625	-0.7627
3.	0.5527	0.3021	-0.6773	0.3737
4.	0.5904	-0.0674	0.3908	0.4175

There is a clear disjuncture between the set of lateral variables and the single apertural one. The tenets of the Perron-Frobenius theorem for square symmetric matrices (Perron, 1907) stipulates that all the components of the vector corresponding to the largest latent root must be positive, in itself a form of constraint. The first latent vector mirrors the influence of the logarithmic constraint - we note that all three such components are of approximately equal value, which is not an unexpected result. The latent vector attached to the second latent root is dominated by the apertural variable.

It is not always appreciated that the interpretation of principal component analyses rests on a built-in artifact of the model. The Perron-Frobenius theorem states unequivocally that among the latent roots of a positive square symmetric matrix there will be a real positive value (the maximum root) the value of which is not surpassed by any other latent root of the matrix, and which has an all-positive latent vector (cf. Zurmühl, 1964, p. 219).

A point of historical interest is well worth noting here. Jacob Bernoulli's burial monument in Basel bears the inscription “iterum renata resurgo” - a reference to the remarkable properties of the logarithmic spiral. The discussion of the special properties of W-curves, to which the logarithmic spiral may be referred, given by Klein (1926. p. 167) is fascinating.

References

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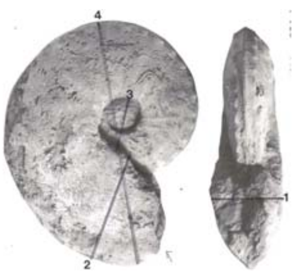

Figure 1: Location of the four distance-measures for the *Schloenbachia* example.