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Exemplification of the logarithmic spiral on the multivariate analysis of planispirally coiled ammonite shells

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Abstract

The logarthmic spiral structure of the conch of coiled ammonites introduces a complication with respect to the multivariate analysis of the variability of certain shell- dimensions by the technique known as "principal components". This is the outcome of the fact that the lateral form of the shell is geometrically (constrained by the properties imposed by the logarithmic spiral. The data used here consist of carefully monitored observations on macroconchs and microconchs of the Albian (Cretaceous) species *Knemiceras persicum* Collignon. The principal components for covariance matrices are interpreted for five "variables" to wit standard distance-measures for the maximum diameter of the conch and four distances recorded for umbilical diameter and three whorl heights. The latent roots and vectors computed for the microconch (males) shells differ slightly, and consistently, from the macroconchs (females). The dominant role of the first component of the first latent vector for both sexes is an expression of the reduced rank of the matrix of covariances.

Introduction and statement of the problem

Ammonite shells are coiled in close approximation to the geometry of the logarithmic spiral. The concepts enunciated in this note require that the shells are not deformed in any manner (tectonic distortion, weathering effects, for example). This imposes a constraint on the results yielded for the shape-analysis of the conch based on measurements made on the shell in lateral aspect if the set of variables are part of the spiral growth pattern (Klein, 1926, pp. 171-173). One of the inescapable outcomes of the extraction of latent roots and vectors of the matrix of covariances of such data is that the square symmetric matrix is not of full rank (i.e. less than the dimensionality of the covariance

matrix *(cf.* Turnbull, 1960, p. 75)). Hence, all possible selections of variables, such as the width of the umbilicus, the maximum diameter of the conch, breadths of whorls are constrained in that they are linearly bound to each other.

(A point of historical interest is worth noting here. Jacob Bernoulli's burial monument in Basel bears the inscription "*iterum renata resurgo*", which is reference to the remarkable properties of the logarithmic spiral. A review of the special properties of W-curves, to which the logarithmic spiral may be referred, is given by Klein (1926. p. 167)).

Sexual dimorphism in ammonites exemplified by *Knemiceras persicum* Collignon (Albian, Iran)

Many, but not all ammonites, display dimorphism, probably an expression of sexual dimorphism according to current interpretations. This condition expresses itself as microconchs (shells of males) and macroconchs (shells of females). Macroconchs are always slightly larger than microconchs.A brief review of purported sexual dimorphism in *Knemiceras persicum* is given in Kennedy et al. (2009).

Although the collections analysed in this note contain many specimens, only few of these qualify for an accurate analysis , due to weathering, deformation of the original proportions and uneven preservation.

Steps in the analysis of macroconchs in relation to microconchs

1. The distance measures are maximum diameter of the conch (Figure 1) are (1), the maximum diameter of the conch in lateral aspect, (2) termination of the last whorl, (3) height at the at the beginning of the last whorl, (4) maximum width of the umbilicus and (5) the whorl-height at 180 degress from the last preserved part of the conch.

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2. The results for **microconchs** (N=17)

The latent roots

1	2	3	4	5
1467.29	58.29	34.66	13.55	3.35

Percentages of the of the latent roots of the trace

1	2	3	4	5
92.91	3.69	2.19	0.86	0.34

Latent Vectors (by columns)

	1	2	3	4	5
1	**0.9296**	0.1299	-0.3088	-0.0455	0.1467
2	0.2291	0.1078	0.7910	-0.5543	-0.0538
3	0.0893	0.3937	0.4584	0.7405	0.2803
4	0.1579	-0.8851	0.2285	0.1880	0.3226
5	0.2246	-0.1818	0.1285	0.3272	-0-8905

The first latent root encompasses almost all of the variabilty (i.e. almost 93% of the trace). With respect to the latent roots the first element of the first latent vector (the maximum diameter of the conch) is unity.

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3. The results for **macroconchs** (N=8)

The latent roots

1	2	3	4	5
2793.54	86.17	25.69	1.90	1.54

Percentages of the latent roots of the trace

1	2	3	4	5
96.04	2.96	0.88	0.065	0.053

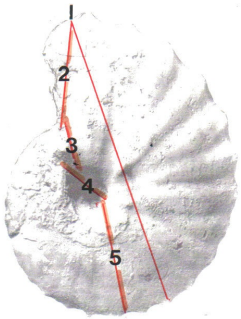
Latent vectors by columns

	1	2	3	4	5
1	**0.9237**	0.1631	-0.2805	-0.1072	0.1734
2	0.2298	0.1132	0.2175	0.8128	-0.4759
3	0.1245	0.1542	0.8177	0.0401	0.5390

4	0.1646	-0.9524	0.0283	0.1826	0.1779
5	0.2269	-0.1723	0.4523	-0.5412	-0.6491

The latent roots and vectors for macroconchs echo those computed for microconchs which in turn are the outcome of the constraint imposed by the geometry of the logarithmic spiral. There is a slight and consistent between the macroconchs and microconchs.

Figure 1: Illustrates the locations of the lateral measurements observed on *Knemiceras persicum* Collignon



Concluding Remarks

The brief note shows that a standard principal component analysis of distances measured on the ammonite conch observed in lateral geometric aspect cannot lead to a statistically meaningful result owing to the intrinsic mathematical properties of the logarithmic spiral (Perron, 1907).

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References

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